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Comment On "On some Properties of Tribonacci Quaternion"

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Abstract

This short commentary serves as a correction of the paper by Akkus and Kızılaslan [I. Akkus and G. Kızılaslan, On some Properties of Tribonacci Quaternion, *An. Şt. Univ. Ovidius Constanţa*, **26**(3), 2018, 5–20].

1 Corrections

Let

$$Q_n = T_n + \mathbf{i}T_{n+1} + \mathbf{j}T_{n+2} + \mathbf{k}T_{n+3},$$

where T_n is the n-th Tribonacci number.

In [1], Akkus and Kızılaslan introduced the following identity (which is part of the identities 6 in [1])

$$Q_n^2 - Q_{n-1}^2 = \widetilde{U}_{n+1}\widetilde{U}_{n-1},\ n \geq 2,$$

where

$$\widetilde{U}_n = U_n + \mathbf{i}U_{n+1} + \mathbf{j}U_{n+2} + \mathbf{k}U_{n+3},$$

with $U_n = T_{n-1} + T_{n-2}$ if $n \geq 2$ and $U_0 = U_1 = 0$.

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However, the identity of this version is wrong. In fact, if $n = 2$, we have

$$\begin{aligned} Q_2^2 - Q_1^2 &= (1 + 2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k})^2 - (1 + \mathbf{i} + 2\mathbf{j} + 4\mathbf{k})^2 \\ &= (-68 + 4\mathbf{i} + 8\mathbf{j} + 14\mathbf{k}) - (-20 + 2\mathbf{i} + 4\mathbf{j} + 8\mathbf{k}) \\ &= -48 + 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}. \end{aligned}$$

and

$$\begin{aligned} \widetilde{U}_3\widetilde{U}_1 &= (2 + 3\mathbf{i} + 6\mathbf{j} + 11\mathbf{k})(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \\ &= -48 - 2\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}. \end{aligned}$$

The correct version should be as follows. For $\widetilde{U}_n$ to be defined as above and $n \in \mathbb{N}$, then

$$Q_n^2 - Q_{n-1}^2 = \widetilde{U}_{n+1}\widetilde{U}_{n-1} + 2\left(T_{-(n+3)}\mathbf{i} - (T_{-(n+2)} - T_{-(n+1)})\mathbf{j} + T_{-(n+2)}\mathbf{k}\right), \tag{1.1}$$

where T_{-n} is the n-th Tribonacci negative number ($n \in \mathbb{N}$) and satisfies the recurrence relation

$$T_{-(n+1)} = \begin{vmatrix} T_n & T_{n+1} \\ T_{n-1} & T_n \end{vmatrix} = T_n^2 - T_{n-1}T_{n+1}, \ n \geq 1.$$

There are several proofs of this famous quaternions, see for example [2, 3]. Actually, this identity plays a central role in proving one of the famous non-commutative properties of Tribonacci quaternions, say

$$\begin{aligned} Q_{n+1}Q_n - Q_nQ_{n+1} &= 2\left(T_{n+3}^2 - T_{n+2}T_{n+4}\right)\mathbf{i} + 2\left(T_{n+1}T_{n+4} - T_{n+2}T_{n+3}\right)\mathbf{j} \\ &\quad + 2\left(T_{n+2}^2 - T_{n+1}T_{n+3}\right)\mathbf{j} \\ &= 2\left(T_{-(n+4)}\mathbf{i} - (T_{-(n+3)} - T_{-(n+2)})\mathbf{j} + T_{-(n+3)}\mathbf{k}\right), \end{aligned} \tag{1.2}$$

with $n \geq 0$.

From Eqs. (1.1) and (1.2), we obtain

$$Q_n^2 - Q_{n-1}^2 - Q_nQ_{n-1} + Q_{n-1}Q_n = \widetilde{U}_{n+1}\widetilde{U}_{n-1}, \ n \geq 1. \tag{1.3}$$

Similarly the identity 5 in [1]. Furthermore, one should note we should use the correct version of the identity (1.2) to obtain the conclusion.

References

[1] I. Akkus and G. Kızılaslan, On some Properties of Tribonacci Quaternion, *An. Şt. Univ. Ovidius Constanţa*, **26**(3), 2018, 5–20.

[2] G. Cerda-Morales, Identities for Third Order Jacobsthal Quaternions, *Adv. Appl. Clifford Algebra*, **27**(2), 2017, 1043–1053.

[3] G. Cerda-Morales, On a Generalization of Tribonacci Quaternions, *Mediter. J. Math.*, **14**(239), 2017, 1–12.

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